

UNCLASSIFIED

AD NUMBER

AD869318

LIMITATION CHANGES

TO:

Approved for public release; distribution is unlimited.

FROM:

Distribution authorized to U.S. Gov't. agencies and their contractors;  
Administrative/Operational Use; FEB 1970. Other requests shall be referred to Rome Air Development Center, Griffiss AFB, NY.

AUTHORITY

RADC ltr 17 Sep 1971

THIS PAGE IS UNCLASSIFIED

869318

RADC-TR-70-51  
Technical Report No. 4  
February 1970



**SIDELOBE-REDUCTION AND INTERFERENCE-SUPPRESSION TECHNIQUES  
FOR PHASED ARRAYS USING DIGITAL PHASE SHIFTERS**

Contractor: Syracuse University  
Contract Number: F30602-68-C-0067  
Effective Date of Contract: 22 September 1967  
Extended to  
22 September 1968  
Contract Expiration Date: 21 September 1970  
Amount of Contract: \$39,947 plus  
\$79,986 for Extension  
Program Code Number: 9E30

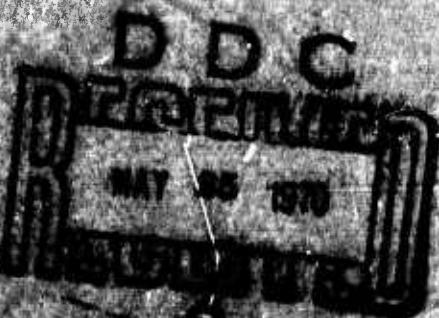
Principal Investigator: Dr. David Cheng  
Phone: Syracuse 476-5541-2651

Project Engineer: Lt. James D. Becker  
Phone: 315-330-4306

Sponsored By  
Advanced Research Projects Agency  
ARPA Order No. 1010

This document is subject to special  
export controls and each transmittal  
to foreign governments or foreign na-  
tionals may be made only with prior  
approval of RADC (EMATS), GAFB,  
N.Y. 12440.

ARPA Air Development Center  
Advanced Systems Division  
Oneida Falls, New York



When US Government drawings, specifications, or other data are used for any purpose other than a definitely related government procurement operation, the government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded, by implication or otherwise, as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

WHITE SECTION <input type="checkbox"/>	DUFF SECTION <input type="checkbox"/>
UNARMED JUSTIFICATION	
BY	
DISTRIBUTION/AVAILABILITY CODE	
SIZE	AVAIL. REG/IR SPECIAL
2	

Do not return this copy. Retain or destroy.

SIDELOBE-REDUCTION AND INTERFERENCE-SUPPRESSION TECHNIQUES  
FOR PHASED ARRAYS USING DIGITAL PHASE SHIFTERS

Dr. David K. Cheng  
Syracuse University

This document is subject to special  
export controls and each transmittal  
to foreign governments or foreign na-  
tionals may be made only with prior  
approval of RADC (EMATS), GAFB,  
NY 13440.

This research was supported by the  
Advanced Research Projects Agency  
of the Department of Defense and was  
monitored by Lt James Becker, RADC  
(EMATS), GAFB, N.Y. 13440 under  
Contract F30602-68-C-0067.

FOREWORD

This report was prepared by Dr. David K. Cheng, Professor of Electrical Engineering, Syracuse University, Syracuse, New York under Contract No. F30602-68-C-0067, ARPA Order No. 1010.

This research was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by RADC Project Engineer Lt. James Becker (EMATS).

Information in this report is embargoed under the U. S. Export Control Act of 1949, administered by the Department of Commerce.

PUBLICATION REVIEW

This technical report has been reviewed and is approved.

James D. Becker  
RADC Project Engineer

## ABSTRACT

Through the introduction of a new parameter the **radiation pattern** of a phased array using digital phase shifters for beam steering can be made periodic and pattern considerations can be confined to a very narrow range of the scan angle. Arrays designed on the basis of a least mean-squared pattern error are shown to require the smallest absolute values of a phase-index function. A systematic and easy-to-apply method is developed for reducing the peak sidelobes of nonreciprocal phased arrays by one-step phase adjustments in certain elements. The amount of possible sidelobe reduction depends on the scan angle and the size of phase-quantization steps. Typical examples for 4-bit phase shifters show that reductions in excess of 9 dB are possible for certain **main-beam directions**. By the appropriate choice of a weighting function, the same technique can be readily used to suppress noise or interference coming from any given direction.

TABLE OF CONTENTS

	PAGE
I. INTRODUCTION-----	1
II. PERIODICITY OF DIGITALLY CONTROLLED ARRAY PATTERNS-----	3
III. THE MINIMUM-PHASE-ERROR CRITERION-----	6
IV. NONRECIPROCAL PHASED ARRAY-----	8
V. PHASE COMPENSATION AND WEIGHTING FUNCTION-----	11
VI. ILLUSTRATIVE EXAMPLES-----	13
VII. CONCLUSION-----	16
VIII. REFERENCES-----	17

## I. INTRODUCTION

Increased attention has recently been directed to the development of phased arrays which utilize digital phase shifters for beam steering. It is well known that a digitally controlled phased array designed on the premise of continuous phase shifts produces undesirable, high sidelobes because of the existence of phase-quantization errors. [1,2] These sidelobes diminish when a large number of bits are used in the digital phase shifters. However, constraints on cost and system complexity can not allow arbitrarily small steps in phase changes. It is therefore important to study the dependence of an array pattern on the quantized phase errors and to search for effective ways for reducing peak sidelobes.

The purpose of this report is threefold. First, it will be shown that by introducing a new parameter in place of the scan angle an array pattern can be made periodic with respect to the new parameter independently of the element spacing or the quantized phase step. This simplifies the study considerably since it will only be necessary to consider the array pattern within a very narrow range of the scan angle. Second, it will be demonstrated that the simple expediency of minimizing the mean-square error of the array-pattern function does not yield uniformly low sidelobes throughout the visible region; some sidelobes may become intolerably high. Third, a new technique will be presented for reducing the peak sidelobes of digitally controlled radar phased arrays. This technique takes advantage of the nonreciprocal nature of latching ferrite phase shifters. It is systematic and does not require random trials.

Examples will show that the peak-sidelobe level obtained on the basis of a minimum squared phase error can be further reduced by 9 or more dB in certain cases. By suitably controlling a weighting function, this technique can be readily used to suppress noise or interference coming from any given direction.

**BLANK PAGE**

## II. PERIODICITY OF DIGITALLY CONTROLLED ARRAY PATTERNS

Consider a linear array of  $2K+1$  elements with uniform spacing  $d$ , as shown in Fig. 1. Let  $I_k$  and  $-2\pi n(k)/M$  denote, respectively, the excitation amplitude and the phase shift in the  $k$ th element, where  $n(k)$  is an integer and  $M=2^i$  for an  $i$ -bit phase shifter. The far field of the array is

$$E(\theta) = \sum_{k=-K}^K I_k \exp\left\{j2\pi\left[\frac{d}{\lambda} \sin \theta - \frac{n(k)}{M}\right]\right\}, \quad (1)$$

where  $\theta$  is measured from the broadside direction. If  $\theta_0$  is the direction of the main beam, (1) can be rewritten as

$$\Sigma(u) = \sum_{k=-K}^K I_k \exp(jku) \exp\left[j \frac{2\pi}{M} f(\alpha, k)\right], \quad (2)$$

where

$$u = \frac{2\pi d}{\lambda} (\sin \theta - \sin \theta_0) \quad (3)$$

$$\alpha = \frac{Md}{\lambda} \sin \theta_0 \quad (4)$$

$$f(\alpha, k) = \alpha k - n(k). \quad (5)$$

From (2), it is clear that the condition  $f(\alpha, k) = 0$  yields the radiation pattern obtained with continuous phase shifts and that the deterioration of the pattern due to quantized phase shifts is uniquely determined by the quantity  $2\pi f(\alpha, k)/M$ . Moreover, since  $\theta_0$  is a single-valued function of  $\alpha$  in the region  $-\pi/2 \leq \theta_0 \leq \pi/2$ ,  $\alpha$ , instead of  $\theta_0$ , can be used to denote the main-beam direction. For convenience, we shall call  $f(\alpha, k)$  a phase-index function.

Let  $-2\pi n_o(\alpha, k)/M$  be the phase shift in the  $k$ th element which produces a pattern pointing in the  $\alpha$ -direction with a prescribed sidelobe level. Then, if we define  $n_o(\alpha+m, k)$  for the new main-beam direction  $\alpha+m$  as

$$n_o(\alpha+m, k) = n_o(\alpha, k) + mk, \quad (6)$$

we have from (5) and (6),

$$f_o(\alpha+m, k) = f_o(\alpha, k). \quad (7)$$

Equation (7) implies that the pattern with beam direction  $\alpha+m$  has the same structure and the same prescribed sidelobe level as that with beam direction  $\alpha$ . In case  $\alpha$  lies in the region  $0 \leq \alpha \leq 1/2$ , we define

$$n_o(1-\alpha, k) = k - n_o(\alpha, k). \quad (8)$$

If the array has a symmetrical amplitude excitation and an antisymmetrical phase distribution; that is, if  $I_k = I_{-k}$  and  $n_o(\alpha, k) = -n_o(\alpha, -k)$ , we obtain, on substituting (8) in (5)

$$\begin{aligned} f_o(1-\alpha, k) &= -f_o(\alpha, k) \\ &= f_o(\alpha, -k). \end{aligned} \quad (9)$$

From (2) and (9), it is seen that the pattern with beam direction  $1-\alpha$  is symmetrical to the pattern with beam direction  $\alpha$  with respect to  $u=0$ . As a result, the consideration of the array radiation pattern can be restricted to the range  $0 \leq \alpha \leq 1/2$ , from which the radiation pattern for any arbitrary value of  $\alpha$  can be derived. The corresponding range in the main-beam direction  $\theta_o$  is, from (4),

$$0 \leq \theta_o \leq \sin^{-1}(\lambda/2Md). \quad (10)$$

For  $M=16$  and  $d=0.7\lambda$ , (10) gives  $0 \leq \theta_0 \leq 2.56^\circ$ , a very small range indeed. Hence,  $\alpha$  represents a useful parameter for specifying the radiation pattern of arrays employing digital phase shifters.

### III. THE MINIMUM-PHASE-ERROR CRITERION

When the phase-index function  $f(\alpha, k) = 0$  in (3), the radiation pattern is identical to that produced by continuous phase shifters and is given by

$$E_o(u) = \sum_{k=-K}^K I_k \exp(jku) . \quad (11)$$

The difference between  $E(u)$  in (2) and  $E_o(u)$  in (11) represents the error in radiation pattern due to quantized phase errors. If  $2\pi f(\alpha, k)/M \ll 1$ , the factor  $\exp[j2\pi f(\alpha, k)/M]$  in (2) can be replaced approximately by  $1 + j2\pi f(\alpha, k)/M$  and the mean-square error of the radiation pattern can be written as

$$\frac{1}{2\pi} \int_0^{2\pi} |E(u) - E_o(u)|^2 du \approx \left(\frac{2\pi}{M}\right)^2 \sum_{k=-K}^K I_k^2 \{f(\alpha, k)\}^2 . \quad (12)$$

Equation (12) indicates that it is necessary to minimize the absolute values of the phase-index function  $f(\alpha, k)$  in order to minimize the mean-square error of the pattern.

As illustrations we choose to compute for a 25-element array ( $K=12$ ) the  $f_m(\alpha, k)$  distributions which have minimum absolute phase-index values for two main-beam directions corresponding to  $\alpha = 1/3$  and  $\alpha = 1/2$ . The results are shown in Fig. 2. These distributions have been used in conjunction with the amplitude excitations necessary to produce a Chebyshev pattern with a -40 dB sidelobe level (if there are no phase errors). Radiation patterns have been computed for  $M=2^4=16$  and are drawn as the

solid curves in Figs. 3 and 4. (The significance of the dashed curves will be discussed in a later section.) It is noted from Fig. 3 that, while most of the sidelobes conform to the designed level, high sidelobes appear in two regions, the highest one being only 20 dB down from the main beam at  $\alpha=1/3$ . The highest sidelobe of the pattern for  $\alpha=1/2$  in Fig. 4 is -23 dB, and the solid curve hardly resembles a Chebyshev pattern. It is evident that the minimum-absolute-phase-error criterion does not yield very good results.

The directions  $u$  where high sidelobes appear do not depend on the value of  $M$ , but the levels of these high sidelobes vary approximately inversely with  $M$ . An examination of the minimum phase-index distribution,  $f_m(\alpha, k)$ , in Fig. 2 reveals the presence of regularity in element phase errors. This regularity can be correlated with both the location and the level of the high sidelobes. A quantum (one-step) change in the digital phase shifter of one or several of the array elements breaks this regularity, but it does not guarantee a sidelobe reduction. A modification of  $f_m(\alpha, k)$  can be found by trial-and-error which will result in some sidelobe reduction, but a trial-and-error approach is impractical for an array with a large number of elements. In the following section, a systematic method will be formulated for peak-sidelobe reduction by taking advantage of the nonreciprocal nature of latching ferrite phase shifters.

#### IV. NONRECIPROCAL PHASED ARRAY

The latching ferrite phase shifters useful in practical applications are inherently nonreciprocal [3], and it is possible to control the phase distribution in a radar array differently for transmitting and for receiving. Hence sidelobe reduction can be effected by suitably adjusting the phase shifts in the elements of an array for transmitting to counteract the high sidelobes in receiving through the introduction of an appropriate weighting function. Let  $E_t(u)$  and  $E_r(u)$  denote the transmitting and the receiving pattern functions respectively, as follows:

$$E_t(u) = \sum_k I_k \exp(jku) \exp[j \frac{2\pi}{M} f(\alpha, k)] \quad (13)$$

$$E_r(u) = \sum_k I_k \exp(jku) \exp[j \frac{2\pi}{M} g(\alpha, k)] , \quad (14)$$

where  $g(\alpha, k)$  is a phase-index function defined in a manner similar to  $f(\alpha, k)$  in (5):

$$g(\alpha, k) = \alpha k - n'(k) , \quad (15)$$

and  $-2\pi n'(k)/M$  is the digital phase shift of the  $k$ th element for the receiving state. When  $g(\alpha, k) = f_m(\alpha, k)$  the mean-square error of the receiving pattern,  $E_r(u) - E_o(u)$ , is a minimum. However, high sidelobes appear in certain directions as pointed out in the preceding section. This drawback can be overcome by a reduction of the sidelobes in the same directions for the transmitting state.

In order to counteract the high sidelobes in the receiving pattern, we consider the following mean-square error of the transmitting pattern:

$$\xi = \frac{1}{2\pi} \int_0^{2\pi} |E_t(u) - E_o(u)|^2 w(u) du , \quad (16)$$

where  $w(u)$  is a nonnegative weighting function which is to have a large value in regions where the receiving pattern has high sidelobes. For symmetrical amplitude and antisymmetrical phase distributions, (16) can be expressed as

$$\xi \approx \left(\frac{2\pi}{K}\right)^2 \sum_{k=1}^K \sum_{p=1}^K a_k a_p v_{kp} \quad (17)$$

with

$$a_k = I_k f(\alpha, k) \quad (18)$$

and

$$v_{kp} = \frac{1}{\pi} \int_0^{2\pi} \sin ku \sin pu w(u) du . \quad (19)$$

It has been assumed in (19) that  $f(\alpha, 0) = 0$ , all phase shifts having been measured from the reference phase at the center element. We now consider the minimization of the mean-square error  $\xi$  by a one-step phase shift in the  $k$ th digital phase shifter away from  $f_m(\alpha, k)$ , the state where the absolute values of the phase-index function is a minimum; that is, we make

$$a_k = I_k [f_m(\alpha, k) + \epsilon_k] , \quad (20)$$

where

$$\epsilon_k = \begin{cases} +1 & \text{for } f_m(\alpha, k) < 0 \\ -1 & \text{for } f_m(\alpha, k) > 0 . \end{cases} \quad (21)$$

When the  $k$ th element is not under phase compensation,  $\epsilon_k = 0$ ; the phase shift in the transmitting state is the same as that in the receiving state. Substitution of (20) in (17) yields

$$\xi \approx \left(\frac{2\pi}{M}\right)^2 \left[ \sum_{k=1}^K \sum_{p=1}^K v_{kp} I_k I_p f_m(\alpha, k) f_m(\alpha, p) + R \right], \quad (22)$$

where

$$R = \sum_{p=1}^K (v_p + \sum_{k=1}^K v_{kp} \epsilon_k) \epsilon_p \quad (23)$$

$$v_p = 2 \sum_{k=1}^K v_{kp} I_k f_m(\alpha, k). \quad (24)$$

For a given main-beam direction  $\alpha$ , the first term in the right-hand side of (22) is determined. The sidelobe-reduction procedure then consists of searching for the one-step phase adjustments in particular elements in order to minimize  $R$ , as expressed in (23).

## V. PHASE COMPENSATION AND WEIGHTING FUNCTION

If a one-step phase adjustment is to be made in the  $k$ th element only in accordance with (20), we have, from (23),

$$R = A_k = \epsilon_k V_k + v_{kk} . \quad (25)$$

It is a very simple matter to find the value of  $k$  which gives a minimum  $R$  for a specified weighting function by numerical computation of  $A_k$  for  $k=1, 2, \dots, K$ . For one-step phase adjustments in  $2N$  elements,  $R$  in (23) can be written as

$$R = \sum_k \sum_p \left\{ \frac{A_k + A_p}{N-1} + 2v_{kp} \epsilon_k \epsilon_p \right\} , \quad (26)$$

where the summations for  $k$  and  $p$  range over the  $N$  different integers denoting the elements whose phase shifters are to be adjusted. The combination of the  $N$  integers which make  $R$  a minimum must be found numerically, but the computation is simple and the procedure straightforward; there is no need to compute any radiation patterns until the particular elements for phase compensation have been selected.

The choice of the weighting function  $w(u)$  for the computation of  $v_{kp}$  in (19) is flexible. The simplest choice is perhaps to make  $w(u)$  zero everywhere except in the regions where high sidelobes appear in the receiving pattern. We assume

$$w(u) = \begin{cases} 1, & u_1 - \Delta u \leq u \leq u_1 + \Delta u \text{ and } u_2 - \Delta u \leq u \leq u_2 + \Delta u \\ 0, & \text{elsewhere} . \end{cases} \quad (27)$$

In (27)  $u_1$  and  $u_2$  denote the locations and  $\Delta u$  the extent of the sidelobes to be reduced. Examination of the solid patterns in Figs. 3 and 4 reveals that it is appropriate to choose  $u_2 = 2\pi - u_1$ . With this choice, we obtain, in substituting (27) in (19),

$$v_{kp} = \frac{2\Delta u}{\pi} \left[ \frac{\sin(k-p)\Delta u}{(k-p)\Delta u} \cos(k-p)u_1 - \frac{\sin(k+p)\Delta u}{(k+p)\Delta u} \cos(k+p)u_1 \right] \quad (28)$$

and

$$v_{kk} = \frac{2\Delta u}{\pi} \left[ 1 - \frac{\sin 2k\Delta u}{2k\Delta u} \cos 2ku_1 \right]. \quad (29)$$

The above expressions are convenient to use in computing  $R$ .

## VI. ILLUSTRATIVE EXAMPLES

The weighting function in (27) has been used to reduce the high sidelobes which appear in the radiation pattern,  $E_r(u)$ , of digitally controlled arrays designed under the minimum-absolute-phase-error criterion in Section III. From Figs. 3 and 4 it is seen that  $u_1 = 2\pi/3$  and  $9\pi/10$  respectively for  $\alpha = 1/3$  and  $1/2$ . An appropriate choice for  $\Delta u$  is  $\pi/10$  for both cases. The essential data are presented in Table I.

TABLE I  
Phase Compensation Data for 25-Element Array

$\alpha$	1/3			1/2			
$w(u)$	$u_1 = 2\pi/3, u_2 = 4\pi/3$ $\Delta u = \pi/10$			$u_1 = 9\pi/10, u_2 = 11\pi/10$ $\Delta u = \pi/10$			
Case	(A)	(B)	(C)	(D)	(E)	(F)	(G)
Pairs of elements compensated (N)	1	2	3	1	2	2	2
Element Nos. (k)	1	4,5	1,8,10	5	3,7	3,9	1,5
R	Value	-0.3301	-0.3783	-0.3791	-0.5503	-0.6549	-0.6193
	Rank	min.	min.	min.	min.	2nd.	3rd.

In case (A) for  $\alpha=1/3$ , only one pair of elements ( $N=1$ ) are phase-compensated, and calculations with (25) show that  $R$  is minimum when  $k=1$ . From Fig. 2(a), it follows that the phase in the first element to the right of the array center should be shifted down by one step and the phase in the first element to the left of the array center ( $k=-1$ ) should be

shifted up by one step on account of the assumed phase antisymmetry. The equivalent pattern defined by

$$E(u) = |E_r(u) E_t(u)|^{1/2} \quad (30)$$

for case (A) is plotted as the solid curve in Fig. 5. In comparison with the solid curve in Fig. 3, it is seen that, although reduced in level by slightly more than 4 dB, the high sidelobes remain visible at about the same locations.

If two pairs of elements are compensated ( $N=2$ ), the phases in elements  $k=4$  and  $k=5$  should be adjusted one step down and up respectively from the  $f_m(1/3, k)$  values shown in Fig. 2(a) in order to obtain a minimum  $R$ . This is listed as case (B) in Table I. The resulting equivalent pattern is plotted as the dashed curve in Fig. 5, which exhibits a further reduction in the levels of the high sidelobes. In case (C), one-step phase adjustments are applied to three pairs of elements ( $N=3$ ). The excitation phases in the 1st, 8th, and 10th elements are shifted down, up, and down by one step respectively for a minimum  $R$ . The equivalent pattern is shown as the dashed curve in Fig. 3. Comparing with the solid curve representing the receiving pattern in the same figure, we see that the highest sidelobe has been reduced from -20 dB to -29 dB, an improvement of 9 dB, and that the low sidelobes in other regions have been raised slightly.

A similar procedure is applied to reduce the high sidelobes in the receiving pattern for  $\alpha=1/2$  in Fig. 4. Four cases are studied using a weighting function  $w(u)$  as specified in (27) and Table I. In case (D),  $N=1$  and  $k=5$ , the phase in the 5th element is shifted down by one step for

minimum R. The equivalent pattern, plotted as the solid curve in Fig. 6, still shows the existence of high sidelobes on both sides of  $u=\pi$ . Case (E) for phase compensation in two pairs of elements ( $N=2$ ) indicates that one-step phase adjustments are required in the 3rd and 7th elements in order to make R minimum. The corresponding equivalent pattern shown dashed in Fig. 6, however, reveals that the original high sidelobes have been over-reduced. At the same time, the sidelobes in some other regions are made unduly high. In an effort to correct this situation, the combinations of 4-element ( $N=2$ ) phase compensation to give the second and third smallest R were examined. These are listed as cases (F) and (G) in Table I. Pattern calculations indicate that case (F), like case (E), still over-reduces the high sidelobes near  $u=\pi$  at the expense of raising the sidelobes in other regions. For this reason the pattern for case (F) is not shown. Case (G) with  $k=1$  and 5 proves to be a good choice, yielding the dashed pattern in Fig. 4. Comparison of the two patterns in Fig. 4 discloses a reduction of 5 dB in the peak-sidelobe level.

Obviously the same technique can be extended to achieve further reduction in sidelobes by phase compensation in more than two pairs of elements. Ultimately it is a compromise between the amount of further improvement and the attendant complexity.

## VII. CONCLUSION

It has been shown that the radiation pattern of a digitally controlled phased array can be made periodic with respect to a new parameter. In so doing, the study of such an array is much simplified and it is only necessary to consider the pattern within a very narrow range of the scan angle. The simple expediency of minimizing the mean-square error of the pattern function does not yield uniformly low sidelobes throughout the visible region. By taking advantage of the nonreciprocal nature of latching ferrite phase shifters, a systematic, easy-to-apply method has been developed for the reduction of peak sidelobes. The same technique can be used to suppress interference or improve signal-to-noise ratio in radar applications. All that is necessary is to choose a weighting function  $w(u)$  which has a large value in the regions (directions) of interference or high noise.

## VIII. REFERENCES

- [1] C. J. Miller, "Minimizing the effects of phase quantization errors in an electronically scanned array," Proc. Symp. on Electronically Scanned Array Techniques and Applications, Report No. RADC-TDR-64-225, vol. 1, pp. 17-38, July 1964.
- [2] N. Goto, M. Sugie and K. Fukumoto, "Radiation patterns of phased arrays using digital phase shifters," Trans. Inst. Electronics Comm. Engrs. (Japan), vol. 52-B, pp. 166-171, March 1969.
- [3] See, for example, J. E. Pippin, "Microwave ferrite device 1968," Microwave J., vol. 11, pp. 29-45, April 1968.

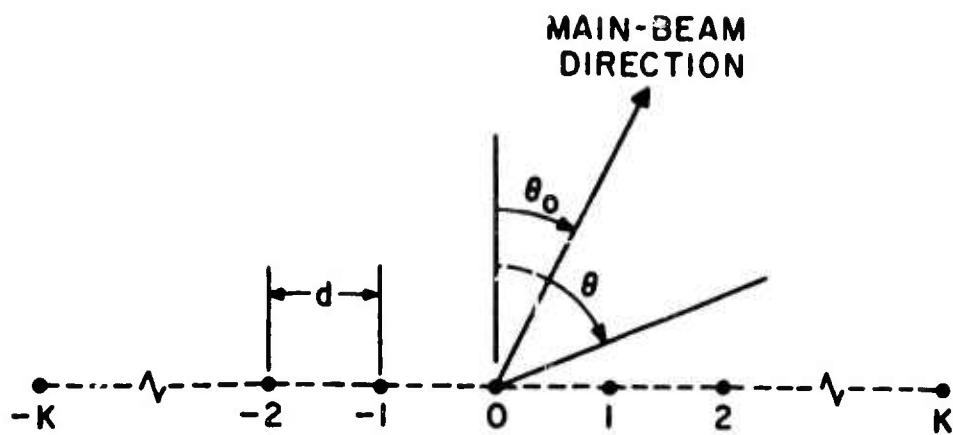
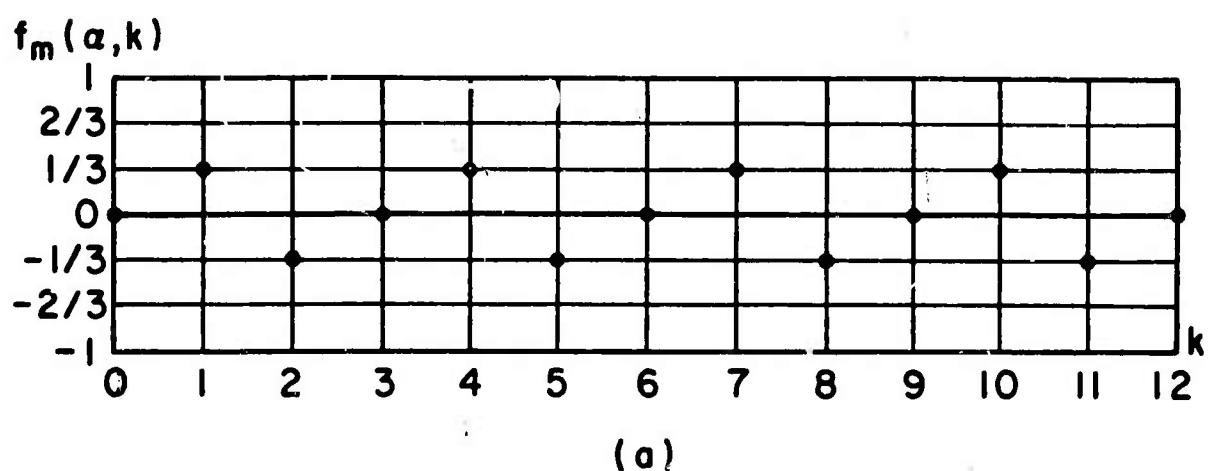
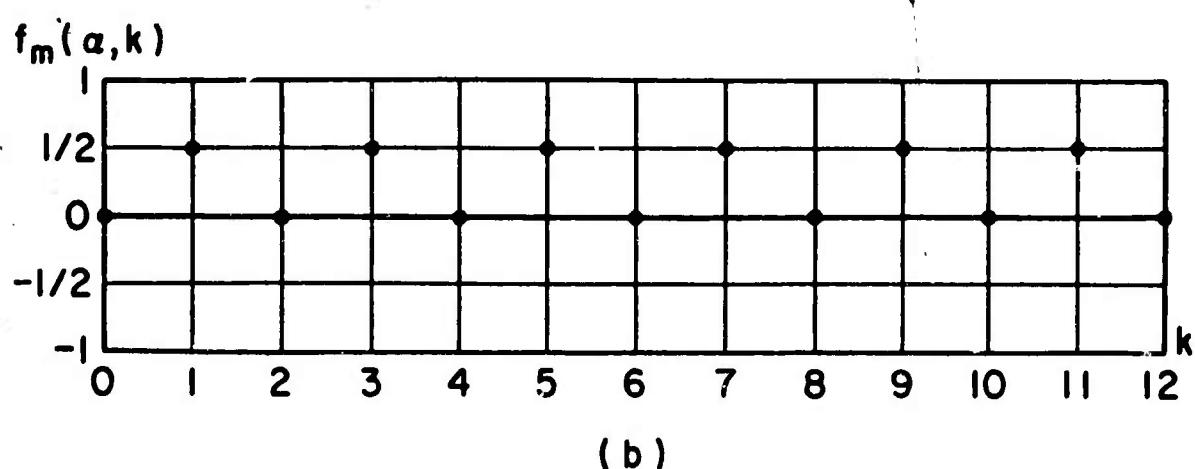


Fig. 1 - A linear array with uniform spacing.



(a)



(b)

Fig. 2 - Minimum phase-index distributions for a 25-element array  
 (a)  $\alpha = 1/3$ , (b)  $\alpha = 1/2$ .

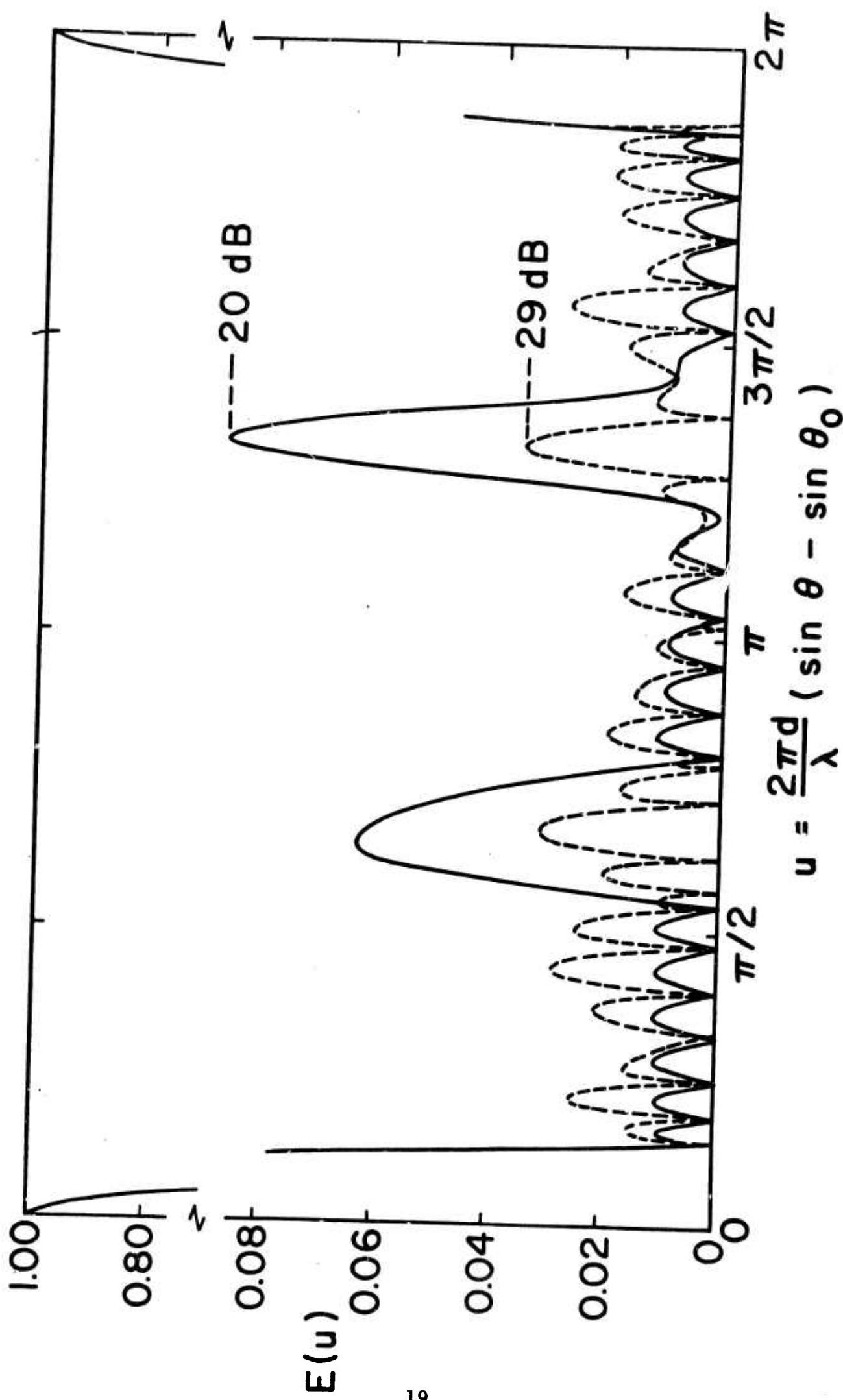


Fig. 3 - Patterns of Chebyshev array with digital phase shifts ( $\alpha=1/3$ )  
 — Minimum-phase-error pattern  
 - - - Case (C),  $N=3$ , using nonreciprocal phase shifters.

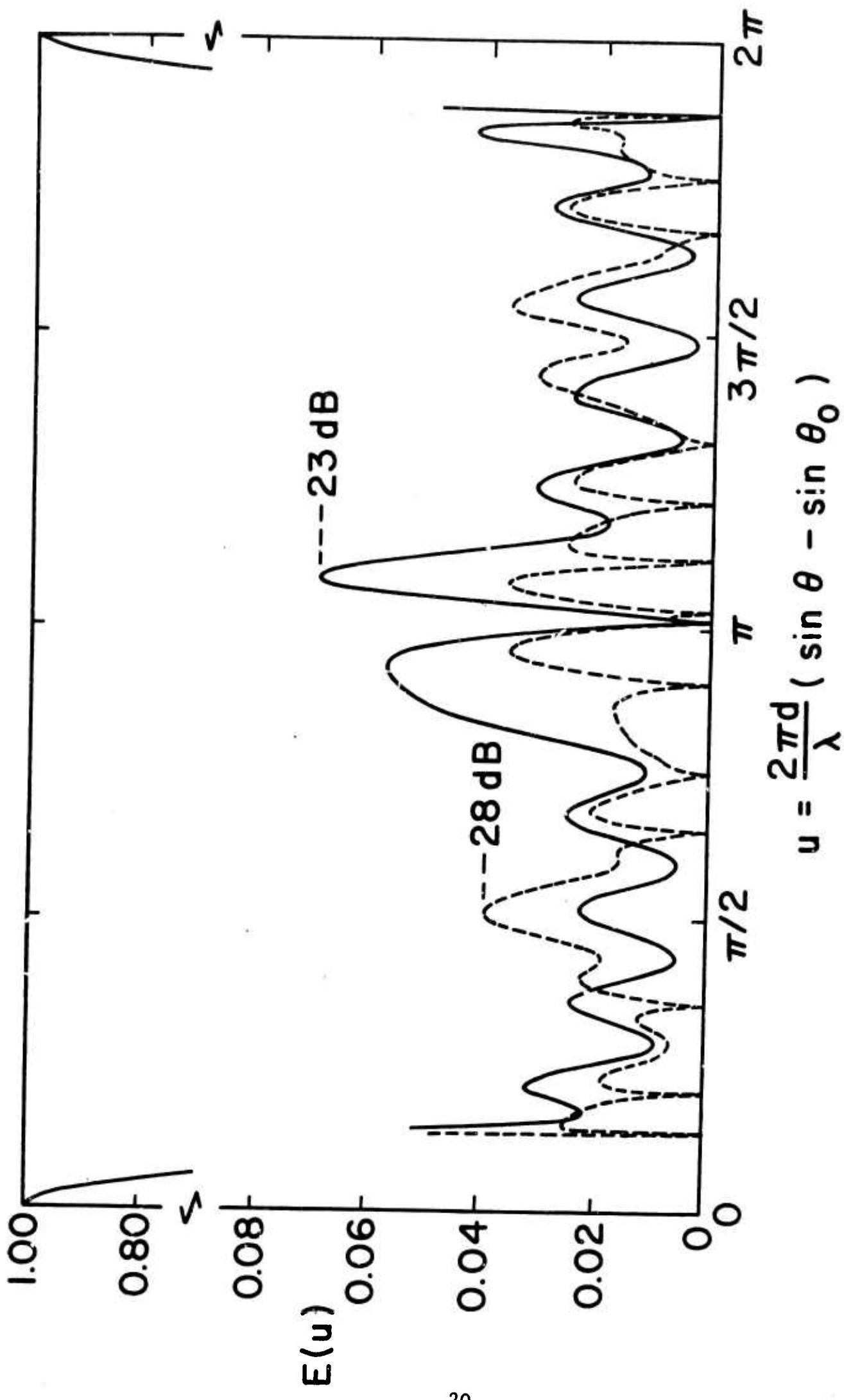


Fig. 4 - Patterns of Chebyshev array with digital phase shifts ( $\alpha=1/2$ )  
 — Minimum-phase-error pattern  
 - - - Case (G),  $N=2$ , using nonreciprocal phase shifters.

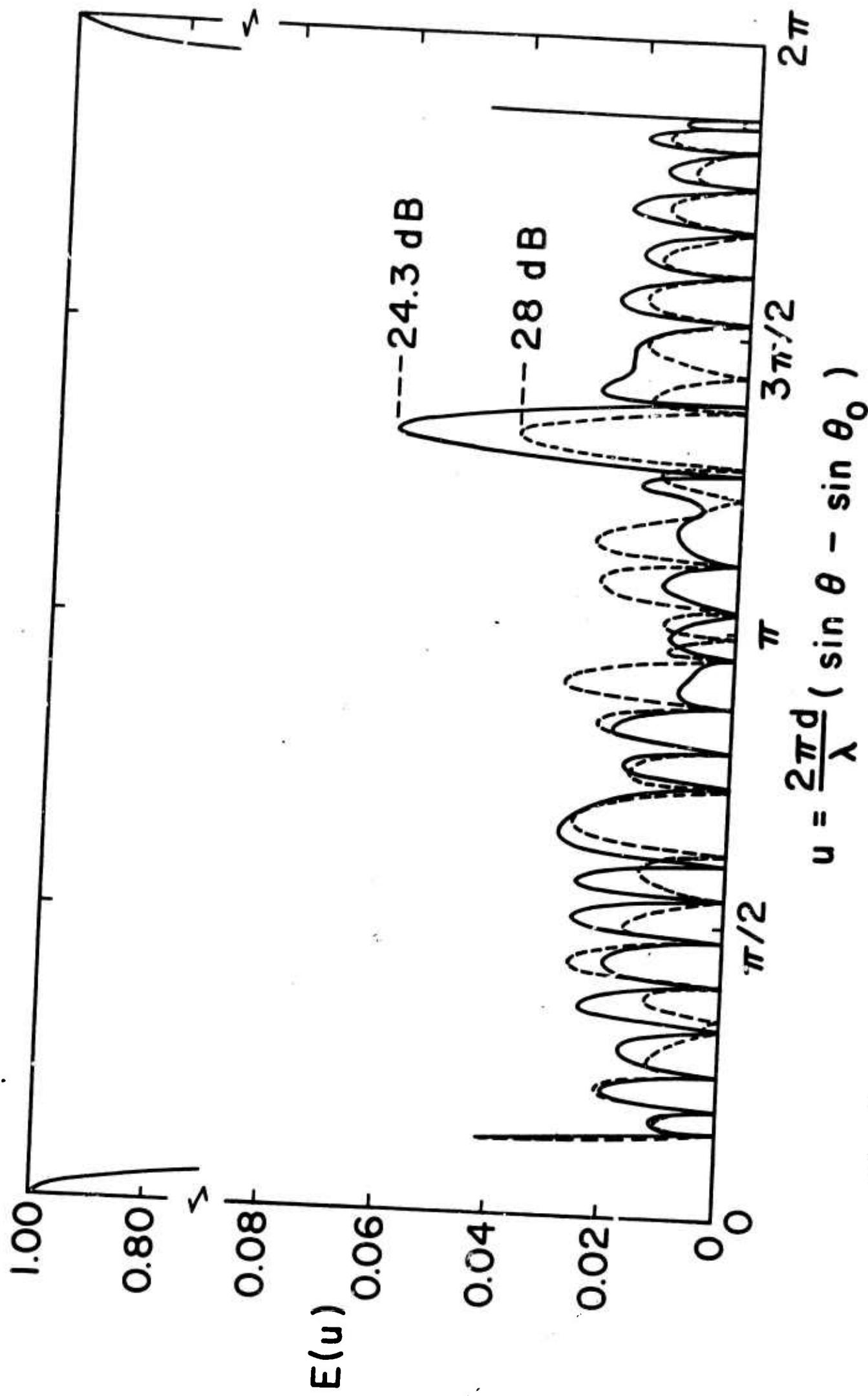


Fig. 5 - Equivalent patterns of nonreciprocal phase-compensated arrays ( $\alpha=1/3$ )

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Phased Arrays Digital Phase Shifts Sidelobe Reduction Interference Suppression						